Math 3

3.6 Natural Logarithms & Base *e*

Unit 3

SWBAT solve logarithmic equations involving natural logs and base e.

Natural base exponential function:

$$y = e^x$$

Natural Logarithm:
logarithm with base *e*

Natural logarithm function:
$y = \ln x$ $y = \log_e x$

Evaluate Natural Base Expressions:
"simply type "2nd e^x (insert number) enter" in your calculator"

a) $e^{0.5}$ 1.6487

b) e^{-8} 0.0003

c) e^{2} 7.3891

d) $e^{-1.3}$ 0.2725

Evaluate Natural Base Expressions:
"simply type "ln (insert number) enter" in your calculator"

a) $\ln 3$ 1.0986

b) $\ln \frac{1}{4}$ -1.3863

c) $\ln 4$ 1.3863

d) $\ln 0.05$ -2.9957

Example 1: Simplify the expression

a) $\frac{\ln e^4}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$

b) $\ln e^{83} = \boxed{83}$

Example 2: Simplify the expression

a) $10 \ln e$ 10

b) $\ln 1$ 0

Example 3: Write each expression as a single natural logarithm.
Use the properties of logs to condense!

a) $3 \ln 5$ $\ln 5^3 = \ln 125$

b) $\ln 24 - \ln 6$ $\ln \frac{24}{6} = \ln 4$

c) $\frac{1}{3}(\ln x + \ln y) - 4 \ln z$ $\ln \frac{\sqrt[3]{xy}}{z^4}$

d) $2 \ln 8 - 3 \ln 4$ $\ln \frac{8^2}{4^3} = \ln \frac{64}{64} = \ln 1 = 0$

Example 4: Solve Base *e* Equations

After isolating the e, use ln on each side to cancel out the e

a) $e^{\frac{x}{4}} + 3 = 9$ $e^{x/4} = 6$ $\log_e 6 = \frac{x}{4}$ $1.7917 = \frac{x}{4}$ $\boxed{x = 7.167}$

b) $5e^{-x} - 7 = 2$ $5e^{-x} = 9$ $e^{-x} = 1.8$ $\log_e 1.8 = -x$ $0.5878 = -x$ $\boxed{x = -0.5878}$

c) $3e^{-2x} + 4 = 10$ $3e^{-2x} = 6$ $e^{-2x} = 2$ $\log_e 2 = -2x$ $0.6931 = -2x$ $\boxed{x = -0.3466}$

d) $e^{3x+1} = e^{13}$ $3x + 1 = 13$ $3x = 12$ $\boxed{x = 4}$

Example 5: Solve Natural Log Equations

After isolating the *ln*, use e on each side to cancel out the *ln*

a) $\ln 5 - \ln x = 4$ $\ln \frac{5}{x} = 4$ $e^4 = 5/x$ $xe^4 = 5$ $x = \frac{5}{e^4}$ $\boxed{x = 0.0916}$

b) $\ln(2m + 3) = 8$ $e^8 = 2m + 3$ $\frac{e^8 - 3}{2} = m$ $\boxed{m = 1488.979}$

c) $\ln \frac{x-3}{4} = 8$ $e^8 = \frac{x-3}{4}$ $4e^8 + 3 = x$ $\boxed{x = 11926.831}$

d) $3 \ln 3x^2 = 1$ $\ln 3x^2 = \frac{1}{3}$ $e^{\frac{1}{3}} = 3x^2$ $x^2 = \frac{e^{1/3}}{3}$ $x^2 = 0.4652$ $\boxed{x = \pm 0.6821}$

Applications of Natural Logs and Base e

***To calculate continuously compounded interest, we use the formula:

$$Y = Pe^{rt}$$

y = ending amt r = rate

P = principal t = time

Example 6: How much money will be in a bank account after 1.5 years if you invested $400 at 7.6% compounded continuously?

$Y = 400e^{(0.076)(1.5)}$

$Y = \$448.30$

Example 7: How much time would it take to triple your principal in an account that pays 6.5% annual interest compounded continuously?

$3 = 1e^{0.065t}$

$\log_e 3 = 0.065t$

$\frac{\ln 3}{0.065} = \frac{0.065t}{0.065}$

$t = 16.9$ years

Practice: Complete the following problems for class work. Show all work.

1. Solve $\ln(14x - 3) = \ln(7x + 11)$

$14x - 3 = 7x + 11$

$7x = 14$

$x = 2$

2. Solve $2e^x - 5 = 1$

$2e^x = 6$

$e^x = 3$

$\log_e 3 = x$

$\ln 3 = x$

$x = 1.0986$

3. $\ln(x - 1) = -2$

$e^{-2} = x - 1$

$e^{-2} + 1 = x$

$x = 1.1353$

4. $\ln(2x - 3) = 2.5$

$e^{2.5} = 2x - 3$

$x = \frac{e^{2.5} + 3}{2} = 7.5912$

5. $\ln 48 - \ln x = \ln 4$

$\ln \frac{48}{x} = \ln 4$

$x = \frac{48}{4} = 12$

6. $e^{3x} \cdot e^x = 15$

$e^{4x} = 15$

$\log_e 15 = 4x$

$\ln 15 = 4x$

$x = \frac{\ln 15}{4} = 0.6776$

Mixed Review: Remember, *all* logarithms share the same rules. Always condense first before solving!

7. $4^{3x} = 12$

$\log_4 12 = 3x$

$x = 0.5975$

8. $\log_6 x + \log_6 9 = \log_6 54$

$\log_6 9x = \log_6 54$

$x = 6$

9. $\log_2 x = -3$

$2^{-3} = x$

$x = 0.125$

10. $\log_2 64 = x$

$x = 6$

11. $\log_2 x - \log_2 5 = 3$

$\log_2 \frac{x}{5} = 3$

$2^3 = \frac{x}{5}$

$x = 40$

12. $\ln 4x + \ln 5 = \ln 20$

$\ln(4x)(5) = \ln 20$

$20x = 20$

$x = 1$

13. Mazie invested $4500 in an account earning 4.3% interest compounded continuously. After how many years will she have $7400 in her account?

$7400 = 4500e^{0.043t}$

$1.6444 = e^{0.043t}$

$\ln 1.6444 = 0.043t$

$t = 11.6$ years